Solving Semi-Linear Systems

A semi-linear system is a system involving the intersection of two functions, one of which is not linear. In this course, the number of solutions to the system can be 0,1,or 2.

To find the number of solutions, combine the two equations and determine the sign of the discriminant ($\Delta = b^2-4ac$).

Three cases:

- 2 solutions $\longrightarrow \Delta > 0$

$y=x^2-6x+11$

$y=2x-4$

$y=y$

$x^2-6x+11=2x-4$

$x^2-8x+15=0$

now check $\Delta=b^2-4ac$

$=(-8)^2-4(1)(15)$

$=4$

$\Delta>0 \Rightarrow$ 2 solutions

Now lets find the solutions.

$x^2-8x+15=0$

$(x-5)(x-3)=0$

$x-5=0 \quad x-3=0$

$x=5 \quad x=3$

now find y:

$x=5;\ y=2x-4$

$=2(5)-4$

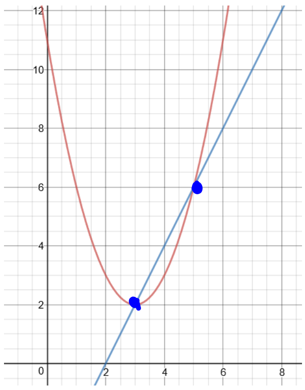
$=6$

$P_1(5,6)$

$x=3;\ y=2(3)-4$

$=2$

$P_2(3,2)$



- 1 solution ⟶ $\Delta = 0$

$y = x^2 - 6x + 11$

$y = 2x - 5$

$y = y$

$x^2 - 6x + 11 = 2x - 5$

$x^2 - 8x + 16 = 0$

gives the number of solutions!

$\Delta = (-8)^2 - 4(1)(16)$

$= 64 - 64$

$= 0 \quad \therefore$ 1 solution

What is the solution?

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

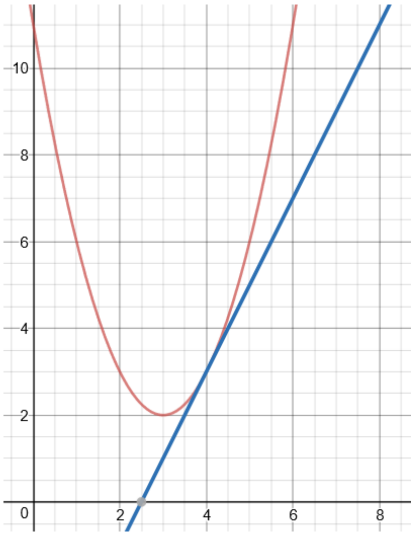
$x - 4 = 0 \Rightarrow \underline{x = 4}$

find y:

$y = 2(4) - 5$

$= 3$

$\underline{P(4,3)}$



- 0 solutions ⟶ $\Delta < 0$

$y = x^2 - 6x + 11$

$y = 2x - 8$

$y = y$

$x^2 - 6x + 11 = 2x - 8$

$x^2 - 8x + 19 = 0$

$\Delta = (-8)^2 - 4(1)(19)$

$= 64 - 76$

$= -12$

$\Delta < 0 \therefore$ NO solutions