Properties of Logarithms Part 2

- Logarithm of a Product

$$\log_a xy = \log_a x + \log_a y$$

- Logarithm of a Quotient

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

- Logarithm of a Power

$$\log_a x^n = n\log_a x$$

Examples:

1) $\log_2(8x4) = \log_2(32) = 5$

$\log_2 8 + \log_2 4 = 3 + 2 = 5$

2) $\log_2 (8/4) = \log_2(2) = 1$

$\log_2 8 - \log_2 4 = 3 - 2 = 1$

3) $\log_2 2^3 = \log_2 8 = 3$

$3\log_2 2 = 3 \times 1 = 3$

$\log_a x^n = n\log_a x$

Write each of the following expressions as a single logarithm.

1) $2\log(x) + 7\log(y) - 3\log(x+1) =$

$\log x^2 + \log y^7 - \log(x+1)^3$

$\log\left(\frac{x^2 y^7}{(x+1)^3}\right)$

2) $2\ln(x) - 3\ln(x^2) - \ln(2x+1) =$

$\ln x^2 - \ln(x^2)^3 - \ln(2x+1)$

$\ln\left(\frac{x^2}{x^6(2x+1)}\right)$

$\ln\left(\frac{1}{x^4(2x+1)}\right)$

3) $\frac{1}{2}\log 4 + \log 2 + 2\log 2$

$\log 4^{\frac{1}{2}} + \log 2 + \log 2^2$

$\log 2 + \log 2 + \log 4$

$\log(2 \times 2 \times 4) = \log 16$

Express the following logarithm as a sum or difference of logarithms.

a) $\log(xy) =$ $\log x + \log y$

b) $\log \frac{xy^2}{4}$ $= \log x + \log y^2 - \log 4$

$= \log x + 2\log y - \log 4$

Given: $x = \log_a 2$, $y = \log_a 3$ and $z = \log_a 7$, express the logarithms in terms of x,y,z.

a) $\log_a 42$ $= \log_a 2 + \log_a 3 + \log_a 7$

$= x + y + z$

b) $\log_a \frac{6}{7}$ $= \log_a 2 + \log_a 3 - \log_a 7$

$= x + y - z$

c) $\log_a 12$ $= \log_a 4 + \log_a 3$

$= \log_a 2^2 + \log_a 3$

$= 2\log_a 2 + \log_a 3$

$= 2x + y$