

Finding the Rule of the Inverse of a Log Function

Given $y = 3\log_2 4(x-1) - 6$ find the inverse.

What kind of function will the inverse be?

Exponential

$$\begin{aligned}
 x &= 3\log_2 4(y-1) - 6 \\
 x+6 &= 3\log_2 4(y-1) \\
 \frac{1}{3}(x+6) &= \log_2 4(y-1) \\
 2^{\frac{1}{3}(x+6)} &= 4(y-1) \\
 \frac{1}{4}(2)^{\frac{1}{3}(x+6)} &= y-1 \\
 \boxed{f^{-1}(x) = \frac{1}{4}(2)^{\frac{1}{3}(x+6)} + 1}
 \end{aligned}$$

$$\begin{aligned}
 \log_c P &= q \\
 \Downarrow \\
 c^q &= P
 \end{aligned}$$

$$\text{Dom } f =]1, +\infty[$$

$$\text{Dom } f^{-1} = \mathbb{R}$$

$$\text{Ran } f = \mathbb{R}$$

$$\text{Ran } f^{-1} =]1, +\infty[$$

$$\therefore \text{dom } f = \text{ran } f^{-1}$$

$$\text{ran } f = \text{dom } f^{-1}$$

Finding the Rule of the Inverse of an Exponential Function

Given: $y = \frac{1}{2}(2)^{\frac{1}{4}(x+3)} - 1$

What kind of function will the inverse be?

Logarithmic

$$\begin{aligned}
 y &= \frac{1}{2}(2)^{\frac{1}{4}(x+3)} - 1 \\
 x &= \frac{1}{2}(2)^{\frac{1}{4}(y+3)} - 1 \\
 x+1 &= \frac{1}{2}(2)^{\frac{1}{4}(y+3)} \\
 2(x+1) &= 2^{\frac{1}{4}(y+3)} \quad \log_c P = a \\
 \log_2 2(x+1) &= \frac{1}{4}(y+3) \quad c^a = P \\
 4 \log_2 2(x+1) &= y+3 \\
 f^{-1}(x) &= 4 \log_2 2(x+1) - 3
 \end{aligned}$$

Dom f : \mathbb{R}

Dom f^{-1} : $]-1, +\infty[$

Ran f : $]-1, +\infty[$

Ran f^{-1} : \mathbb{R}

$\therefore \text{dom } f = \text{ran } f^{-1}$

$\text{ran } f = \text{dom } f^{-1}$