Graphing Logarithmic Functions in the forms:

$\boxed{f(x)= a\log_c b(x-h) + k}$ or $\boxed{f(x) = \log_c \pm(x-h) + k}$

Simplified form

Vertical Asymptote: $\boxed{x = h}$

Variation:

Increasing

- $c > 1$, *a* and *b* have the same sign or *b* is positive in simplified form
- $0 < c < 1$, *a* and *b* have opposite signs or *b* is negative in simplified form

Decreasing

- $0 < c < 1$, *a* and *b* have the same sign or *b* is positive in simplified form
- $c > 1$, *a* and *b* have opposite signs or *b* is negative in simplified form

Orientation of the curve:

- If *b* is positive, the curve is to the right of the asymptote.
- If *b* is negative, the curve is to the left of the asymptote.

Examples: Graph the following

1. $f(x) = \log_4(x - 4) - 1$

- Asymptote: $x = 4$
- Increasing, since $c > 1$, b is positive
- b is positive, so curve is to the right of the asymptote

Now find two points.

1) Zero: $\log_4(x-4) - 1 = 0$

$$\log_4(x-4) = 1$$

$$x - 4 = 4^1$$

$$x - 4 = 4$$

$$x = 8$$

$$P(8,0)$$

2) y-intercept (does not exist since curve is to the right of the asymptote)

So...make x = 20.

$$f(x) = \log_4(20-4) - 1$$

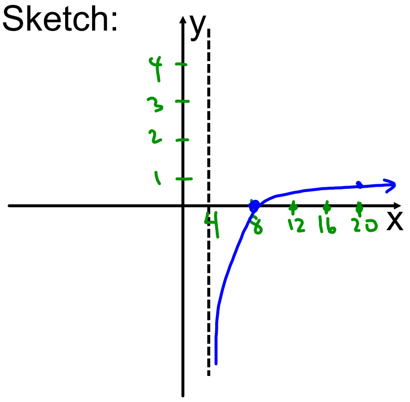
$$= \log_4(16) - 1$$

$$= 2 - 1$$

$$= 1$$

$$P_2(20,1)$$

Sketch:



Domain: $]4, \infty[$

Range: $\mathbb{R}$

Positive: $[8, +\infty[$

Negative: $]4, 8]$

2. $f(x) = -2\log_4 5(x+3) + 1$

- Asymptote: $x = -3$
- Decreasing since $c > 1$, and a and b have opposite signs.
- b is positive, so to the right of the asymptote.

Now we need two points.

Zero: $-2\log_4 5(x+3) + 1 = 0$

$-2\log_4 5(x+3) = -1$

$\log_4 (5(x+3)) = \frac{1}{2}$

$5(x+3) = 4^{\frac{1}{2}}$ $\quad (4^{\frac{1}{2}} = \sqrt{4} = 2)$

$5(x+3) = 2$

$x + 3 = \frac{2}{5}$

$x = -2.6$

$(-2.6, 0)$

y-intercept:

$y = -2\log_4 5(0+3) + 1$

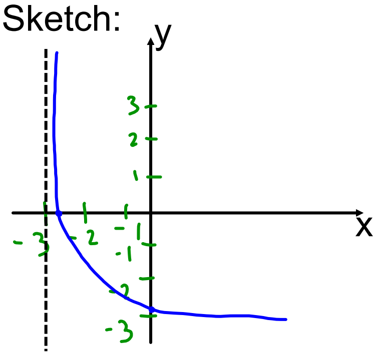
$= -2\log_4 15 + 1$

$= -2\left(\frac{\log 15}{\log 4}\right) + 1$

$= -2.91$

$(0, -2.91)$

Sketch:



Domain: $]-3, \infty[$

Positive: $]-3, -2.6]$

Negative: $[-2.6, \infty[$