Ursula Ophelia Munny (U.O. Munny, for short) owes her bank $100.

Ms. Munny's bank charges 10% interest on debts every month. If she doesn't pay, how much will Ms. Munny owe her bank after 1 month? After 2 months?

$100 + 10\% = 100 + 0.1(100) = 100 + 10 = 110$

$110 + 10\% = 110 \cdot 1.1 = 121$

After a year, Ms. Munny will owe $313.84. How come?

$100 \underbrace{\cdot 1.1 \cdot 1.1 \cdots}_{\text{12 times}} = 100(1.1)^{12} = 313.84$

Exponential Functions

Keywords: doubles, triples, halves, % increase/decrease.
Applications: population, interest, debt, bacteria, chemicals...

$f(x)=ac^x$ or $f(x)=ac^{bx}$

$100(1.1)^x$

a = initial amount

c = rate of increase/decrease*

b = number of times *c* is applied in a given time period

*Note: c can be given as a number, or as a decimal/ percentage:

Number: doubles (c=2), triples (c=3), halves (c=1/2)

Percentage: 1±% increase/decrease:

- A debt increases by 10% each month:
  c=1+10%=1+0.1=1.1
- A car loses 5% of its value each month:
  c=1-5%=1-0.05=0.95

Examples: identify x, y, a, b, c, and write the rule

1. A petri dish has 50 bacteria initially. The number of bacteria triples every hour.

x: time (hours)
y: number of bacteria

a= 50
b= 1
c= 3

$\Rightarrow f(x) = 50(3)^{1x} = 50(3)^x$

2. A frog pond has an initial population of 25 frogs. The population quadruples twice a year.

x: time (years)
y: number of frogs

a= 25
b= 2
c= 4

$\Rightarrow f(x) = 25(4)^{2x}$

3. There are initially 100 bacteria in a sample. The bacteria double every 20 minutes.

x: time (hours)
y: number of bacteria

a= 100
b= 3
c= 2

$\Rightarrow f(x) = 100(2)^{3x}$

4. An initial population of 2000 penguins increases by 18% every year.

x: time (years)
y: number penguins

a= 2000
b= 1
c= 1.18

$\Rightarrow f(x) = 2000(1.18)^x$

5. A car is purchased for $40,000 and loses 17% of its value every year.

$1 - \frac{17}{100} =$

x: Time (year)
y: value of the car

a= 40,000
b= 1
c= 0.83

$1 - 0.17 = 0.83$

$\Rightarrow f(x) = 40{,}000(0.83)^x$

6. An initial population of 150 hippos increases by 6% every 2 years.

x: time (years)
y: number of hippos

a= 150
b= 1/2
c= 1 + 0.06 = 1.06

$\Rightarrow f(x) = 150(1.06)^{\frac{1}{2}x}$