

Piecewise Functions

A **piecewise function** is a function that is defined on a sequence of intervals.

Ex.1: the rule of a piecewise function is shown below:

$$f(x) = \begin{cases} -\frac{1}{x+5} + 2 & \text{if } x \in]-\infty, -4] \\ -\sqrt{x+4} + 1 & \text{if } x \in [4, 5] \\ 1.5|x-5| - 2 & \text{if } x \in [5, +\infty[\end{cases}$$

$x = -4$
 $\therefore y = 1$

Determine the:

a) Domain

$\mathbb{R} \setminus \{5\}$

b) Range

\mathbb{R}

c) Initial value

$y = -\sqrt{0+4} + 1 = -2 + 1 = -1$
 $(0, -1)$

d) Zeros

$0 = \frac{-1}{x+5} + 2$
 $-2(x+5) = -1$
 $x+5 = \frac{1}{2}$
 $x = \frac{1}{2} - 5 = -\frac{9}{2}$

$0 = -\sqrt{x+4} + 1$
 $1 = \sqrt{x+4}$
 $1 = x+4$
 $x = -3$

$0 = 1.5|x-5| - 2$
 $\frac{4}{3} = |x-5|$
 $x-5 = \pm \frac{4}{3}$
 $x = 5 \pm \frac{4}{3}$
 $x = \frac{19}{3}$

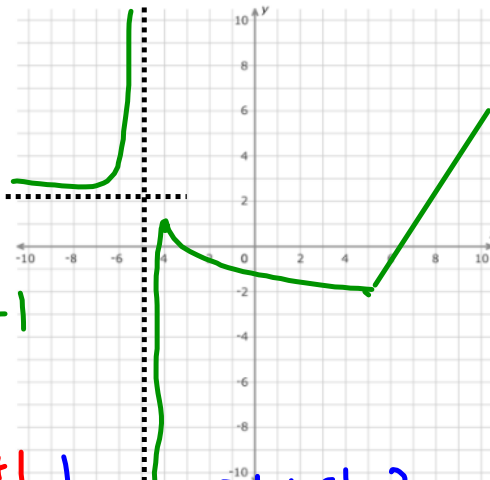
$x-5 = \frac{4}{3}$
 $x = \frac{19}{3}$

e) Sign

$+$: $]-\infty, -5[\cup]-\frac{9}{2}, -3[\cup]\frac{19}{3}, +\infty[$
 $-$: $]-5, -\frac{9}{2}] \cup]-3, \frac{19}{3}]$

f) Variation

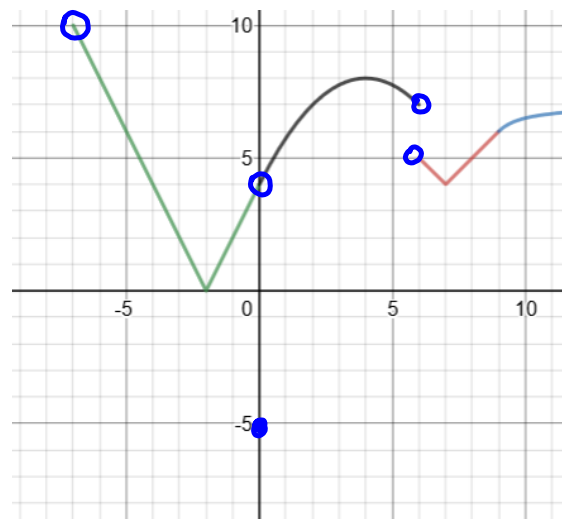
inc.: $]-\infty, -5[\cup]-5, -4] \cup [5, +\infty[$
dec.: $[-4, 5]$



Ex.2: Graph the given piecewise function, then determine the following:

$$f(x) = \begin{cases} 2|x + 2| & \text{if } -7 < x < 0 \\ -5 & \text{if } x = 0 \\ -\frac{1}{4}(x - 4)^2 + 8 & \text{if } 0 < x < 6 \\ |x - 7| + 4 & \text{if } 6 < x < 9 \\ -\frac{1}{x - 8} + 7 & \text{if } x \geq 9 \end{cases}$$

a) Domain
 $] -7, 6[\cup] 6, +\infty[$
 b) Range
 $\{-5\} \cup] 0, 10[$
 c) Initial value
 $f(0) = -5$



d) Zeros
 $x = -2$

e) Sign
 $f(x) > 0:] -7, 0[\cup] 0, 6[\cup] 6, +\infty[$
 $f(x) \leq 0: x \in \{-2, 0\}$
 f) Variation
 increasing: $[-2, 0[\cup] 0, 4] \cup] 7, +\infty[$
 decreasing: $] -7, -2] \cup [4, 6[\cup] 6, 7]$