

Quadratic Function in Factored Form

$$f(x) = a(x-x_1)(x-x_2)$$

→ where x_1 and x_2 are the zeros of the function

Given $f(x) = 2x^2 - 7x + 3$

Change into the form $f(x) = a(x-x_1)(x-x_2)$

$a = 2$

Now find the zeros by factoring.

$$2x^2 - 7x + 3 = 0$$

$$(2x - 1)(x - 3) = 0$$

$$2x - 1 = 0 \quad x - 3 = 0$$

$$2x = 1$$

$$x_2 = 3$$

$$x_1 = 1/2$$

$$\therefore f(x) = 2(x - 1/2)(x - 3)$$

Note: if there is only one zero, $x_1 = x_2 = 4$, for example, and $a = 3$, the equation becomes:

$$f(x) = 3(x-4)^2.$$

Example 1:

Find the rule of the parabola with two zeros x_1 and x_2 passing through the point $P(3,2)$.

$$f(x) = a(x - x_1)(x - x_2)$$

$$f(x) = a(x + 1)(x - 2) \quad \text{Sub in zeros}$$

$$2 = a(3 + 1)(3 - 2)$$

$$2 = a(4)(1)$$

$$2 = \frac{4a}{1}$$

$$a = \frac{1}{2}$$

$$f(x) = \frac{1}{2}(x + 1)(x - 2)$$

Now, change the rule into general form.

Expand: $f(x) = \frac{1}{2}(x + 1)(x - 2)$

$$= \frac{1}{2}(x^2 - x - 2)$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{2}x - 1$$

Now put it in Standard Form:

Vertex: $h: x = \frac{-b}{2a} = \frac{-(-\frac{1}{2})}{2(\frac{1}{2})} = \frac{1}{2}$

k: $f(\frac{1}{2}) = \frac{1}{2}(\frac{1}{2})^2 - \frac{1}{2}(\frac{1}{2}) - 1$

$$= -\frac{1}{8}$$

$$= -\frac{9}{8}$$

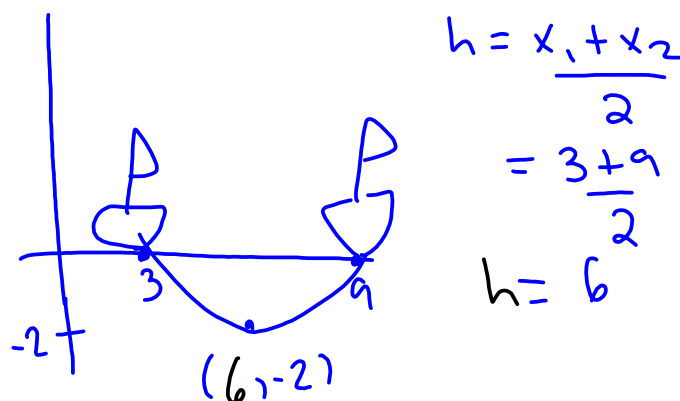
$$\therefore f(x) = \frac{1}{2}\left(x - \frac{1}{2}\right)^2 - \frac{9}{8}$$

Example 2:

The dive of a submarine is recorded and graphed on a Cartesian plane, with a scale in km. The submarine submerges at 3 km and resurfaces at 9 km. If the maximum depth reached is 2 km below the surface of the water, what is the equation of the submarine's trajectory, assuming it is parabolic in shape?

$$x_1 = 3 \quad x_2 = 9$$

$$k = -2$$



$$f(x) = a(x - x_1)(x - x_2)$$

$$f(x) = a(x - 3)(x - 9)$$

$$-2 = a(6 - 3)(6 - 9)$$

$$-2 = a(3)(-3)$$

$$-2 = \frac{a(-9)}{-9}$$

$$a = \frac{2}{9}$$

$$f(x) = \frac{2}{9}(x - 3)(x - 9)$$