Example 1:

$\begin{cases} 2x + y = 7 \\ 4x - y = 5 \end{cases}$

$2x+y=7$
$+\ 4x-y=5$
$6x = 12$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$

$2x+y=7$
$2(2)+y=7$
$4+y=7$ $\cdot -4$
$y=3$

$(2,3)$

OR

$\begin{cases} (2x+y=7)\cdot -2 \\ 4x-y=5 \end{cases}$

$-4x-2y=-14$
$+\ 4x-y=5$
$-3y=-9$

$\frac{-3y}{-3} = \frac{-9}{-3}$

$y=3$

$2x+(3)=7$
$2x=7-3$
$x=2$
$(2,3)$

Example 2:

$\begin{cases} 3x - 2y = 9 \\ 6x - y = 27 \end{cases}$ $\cdot -2$

$-6x+4y=-18$
$+\ 6x-y=27$
$3y=9$

$\frac{3y}{3} = \frac{9}{3}$

$y=3$

$6x-y=27$
$6x-3=27$
$6x=30$
$x=5$

$(5,3)$

Example 3:

$\begin{cases} -x + 2y = 2 \\ 5x - y = 8 \end{cases}$ $\cdot 2$

$-x + 2y = 2$

$+ \ 10x - 2y = 16$

$9x = 18$

$x = 2$

$5x - y = 8$

$5(2) - y = 8$

$10 - y = 8$ -10

$-y = -2$

$y = 2$

$(2, 2)$

Example 4:

$\begin{cases} 5x + 7y = 22 \\ 2x - 3y = 3 \end{cases}$ $\cdot -2$, $\cdot 5$

$-10x - 14y = -44$

$+ \ 10x - 15y = 15$

$-29y = -29$

$\frac{-29y}{-29} = \frac{-29}{-29}$

$y = 1$

$2x - 3y = 3$

$2x - 3(1) = 3$

$2x - 3 = 3$

$2x = 6$

$x = 3$

$(3, 1)$

Example 5:

$\begin{cases} 2x - 3y = 2 \\ 3x + 2y = 29 \end{cases}$ $\cdot 3$, $\cdot -2$

$6x - 9y = 6$

$-6x - 4y = -58$

$-13y = -52$

$\frac{-13y}{-13} = \frac{-52}{-13}$

$y = 4$

$2x - 3(4) = 2$

$2x - 12 = 2$

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

$(7, 4)$

Word Problems with Elimination Method

Chris and Amy are selling pies for a school fundraiser. Customers can buy apple pies and lemon meringue pies. Chris sold 6 apple pies and 4 lemon meringue pies for $96. Amy sold 6 apple pies and 5 lemon meringue pies for 108$. What is the cost of each pie? (x, y)

1) Define the variables

x: cost of an apple pie

y: cost of lemon meringue pie

2) Write the linear system:

$$\begin{cases} 6x+4y=96 \\ 6x+5y=108 \end{cases} \cdot -1$$

3) Solve!

$$\begin{array}{r} -6x-4y=-96 \\ +\ 6x+5y=108 \\ \hline y=12 \end{array}$$

$$\begin{aligned} 6x+5y &= 108 \\ 6x+5(12) &= 108 \\ 6x+60 &= 108 \\ 6x &= 48 \\ x &= 8 \end{aligned}$$

1 apple pie costs $8.

1 lemon meringue pie costs $12.

Ron and Jake are selling blueberry and chocolate chip muffins for the fundraiser. Ron sold 8 blueberry muffins and 12 chocolate chip muffins for $92. Jake sold 2 blueberry muffins and 4 chocolate chip muffins for 28$. What is the price of each muffin?

1) Variables

x: $ of blueberry muffin

y: $ of choc. chip. muffin.

2) System

$$\begin{cases} 8x+12y=92 \\ (2x+4y=28)\times -4 \end{cases}$$

$$\begin{array}{r} 8x+12y=92 \\ +\ -8x-16y=-112 \\ \hline -4y=-20 \\ \frac{-4y}{-4}=\frac{-20}{-4} \\ y=5 \end{array}$$

$$\begin{array}{l} 2x+4y=28 \\ 2x+4(5)=28 \\ 2x+20=28 \\ 2x=8 \\ x=4 \\ (4,5) \end{array}$$

The cost of a blueberry muffin is $4.

Choc. chip muffin is $5.

Elimination word problems

1. A school is selling tickets to the annual talent show. On the first day of ticket sales, the school sold 4 senior citizen tickets and 5 student tickets for a total of $102. On the second day, the school sold 8 senior citizen tickets and 5 student tickets for a total of $134. What is the price of each ticket?

x: cost of senior c. ticket
y: cost of student ticket

$\begin{cases} 4x+5y=102 \\ 8x+5y=134 \end{cases}$ $\times -1$

$-4x-5y=-102$
$+\ 8x+5y=134$
$4x = 32$
$x = 8$

$x=8, y=?$
$4x+5y=102$
$4(8)+5y=102$
$5y=70$
$y=14$

Senior tickets sell for $8.
Student tickets go for $14.

2. Julie and Marcus are selling tickets for their dance school's winter show. Julie sells 3 regular and 5 senior citizen tickets for a total of $70. Marcus earned $216 from selling 12 regular tickets and 12 senior citizen tickets. Find the price of a senior citizen ticket and the price of a regular ticket.

x: # reg. tickets
y: # senior tickets

$\begin{cases} 3x+5y=70 \\ 12x+12y=216 \end{cases}$ $\times -4$

$-12x-20y=-280$
$+\ 12x+12y=216$
$-8y=-64$
$y=8$

$y=8, x=?$
$3x+5y=70$
$3x+5(8)=70$
$3x=30$
$x=10$

Price of senior ticket: $8.
reg. ticket: $10.

3. Khadija's school is selling tickets for the school play, at a price of $5 for students and 13$ for non-students. In total, they raised $3403 from both types of tickets. Khadija knows that, in total, 407 tickets were sold. How many student tickets were sold? How many non-student tickets were sold?

x: number of stud. tickets
y: # of non-stud. tickets

$$\begin{cases} 3403 = 5x + 13y \\ 407 = x + y \end{cases} \quad (407 = x + y) \times -5$$

$$+ \quad 3403 = 5x + 13y$$
$$-2035 = -5x - 5y$$
$$1368 = 8y$$
$$171 = y$$

$y = 171, \; x = ?$

$407 = x + 171$
$407 - 171 = x$
$x = 236$

They sold 236 student tickets and 171 non-student tickets.

4. The senior classes at Heritage and Centennial planned separate trips to New York City. The senior class at Heritage rented and filled 16 vans and 5 buses with 417 students. Centennial rented and filled 10 vans and 8 buses with 480 students. How many students can a van carry? How many students can a bus carry?

x: # of students in a van.
y: # of students in a bus.

$$\begin{cases} 16x + 5y = 417 \\ 10x + 8y = 480 \end{cases} \quad (16x + 5y = 417) \times 5 \quad (10x + 8y = 480) \times -8$$

$$80x + 25y = 2085$$
$$+ \quad -80x - 64y = -3840$$
$$-39y = -1755$$
$$y = 45$$

$y = 45, \; x = ?$

$10x + 8y = 480$
$10x + 8(45) = 480$
$10x = 120$
$x = 12$

A bus fits 45 people
A van fits 12 people.