Quadratic Equation in General Form

$$f(x) = ax^2 + bx + c$$

Given: $f(x) = 2(x-1)^2 - 3$, change it into general form.

Solution: Expand!

$$f(x) = 2(x-1)(x-1) - 3$$
$$= 2(x^2 - 2x + 1) - 3$$
$$= 2x^2 - 4x + 2 - 3$$
$$= 2x^2 - 4x - 1 \leftarrow GF$$

Vertex: $V\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ $\quad h \quad k$

$x = \frac{-b}{2a}$

Axis of Symmetry: $x = \frac{-b}{2a}$ or $x = h$

The sign of "a" still determines if the parabola opens up (+) or down (-).

Finding the Zeros

There are three cases, depending on the sign of the discriminant, $\Delta = b^2 - 4ac$.

Remember: The discriminant comes from the quadratic formula, which can be used to solve quadratic equations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case 1: $\Delta > 0$ ⟶ 2 solutions

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} \qquad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Case 2: $\Delta = 0$ ⟶ 1 Solution

$$x = \frac{-b}{2a}$$

Case 3: $\Delta < 0$ ⟶ No Solutions

Examples: Find the zeros, if they exist, of the following quadratic functions.

a) $f(x) = 2x^2 + 3x - 2$

$a = 2$, $b = 3$, $c = -2$

$\Delta = b^2 - 4ac = (3)^2 - 4(2)(-2) = 25$ $\Delta > 0 \therefore$ 2 sols

$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-3 - \sqrt{25}}{2(2)} = \frac{-3-5}{4} = -2$

$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-3 + \sqrt{25}}{2(2)} = \frac{-3+5}{4} = \frac{1}{2}$

$S = \{-2, \frac{1}{2}\}$

b) $f(x) = 4x^2 + 12x + 9$

$a = 4$, $b = 12$, $c = 9$

$\Delta = b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0 \therefore$ 1 sol.

$x = \frac{-b}{2a} = \frac{-12}{2(4)} = -\frac{12}{8} = -\frac{3}{2}$

$S = \{-\frac{3}{2}\}$

c) $f(x) = -x^2 + x - 1$

$a = -1$, $b = 1$, $c = -1$

$\Delta = b^2 - 4ac = (1)^2 - 4(-1)(-1) = 1 - 4 = -3 \therefore$ no sols. $\Delta < 0$