

Graphing Parabolas

$$\text{Graph: } f(x) = -\frac{1}{2}(x-1)^2 + 2$$

Method:

1. State the Vertex: $V(h,k)$
2. Determine the opening based on the sign of "a"
3. Find the zeros if they exist
4. Find the initial value ($x=0, y=?$)
5. Make a table of values

Solution: $f(x) = -\frac{1}{2}(x-1)^2 + 2$

1. $V(1, 2)$

2. $a = -\frac{1}{2} \cap$

3. Check: $-\frac{k}{a} = -\frac{2}{-\frac{1}{2}} = 4 \quad a > 0 \therefore \text{open up.}$

$$\begin{aligned} x_1 &= 1 - \sqrt{4} & x_2 &= 1 + \sqrt{4} \\ &= 1 - 2 & &= 1 + 2 \\ &= -1 & &= 3 \end{aligned}$$

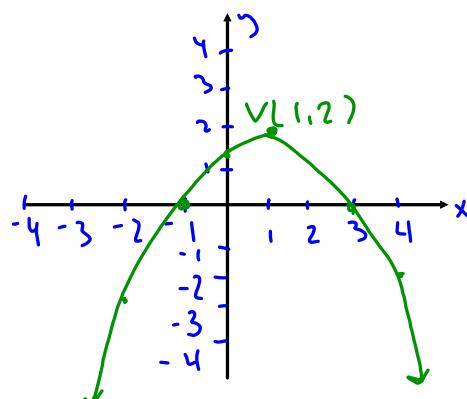
4. $x=0, y=?$

$$\begin{aligned} y &= -\frac{1}{2}(0-1)^2 + 2 \\ &= -\frac{1}{2}(-1)^2 + 2 \\ &= 1.5 \end{aligned}$$

P(0, 1.5)

5.

x	-2	-1	0	1	2	3	4
	-2.5	0	1.5	2	0	-2.5	



Finding the Rule of a Parabola given the Vertex and a PointExample 1:

Given $V(1, -2)$ and a point $P(3, 4)$ find the rule of the parabola in standard form. $f(x) = a(x-h)^2 + k$

Method:

1. Plug in h, k
2. Find a using $P(x, y)$

Solution:

$$\begin{aligned} f(x) &= a(x-h)^2 + k \\ f(x) &= a(x-1)^2 - 2 \quad V(1, -2) \\ 4 &= a(3-1)^2 - 2 \quad P(3, 4) \\ b &= a(2)^2 \\ \frac{b}{4} &= \frac{a(4)}{4} \Rightarrow a = \frac{b}{4} = \frac{3}{2} \\ \therefore f(x) &= \frac{3}{2}(x-1)^2 - 2 \end{aligned}$$

If $f(x) = 22$, what are the possible values of x ?

$$\begin{aligned} 4 &= 22, x = ? \\ 22 &= \frac{3}{2}(x-1)^2 - 2 \\ 24 &= \frac{3}{2}(x-1)^2 \\ \frac{3}{2} &\quad \frac{3}{2} \\ 16 &= (x-1)^2 \quad (\sqrt{\text{both sides}}) \\ \pm \sqrt{16} &= \sqrt{(x-1)^2} \end{aligned}$$

$$\begin{aligned} \pm 4 &= x-1 \\ -4 &= x-1 \quad \text{or} \quad 4 = x-1 \\ -3 &= x \quad 5 = x \\ \therefore x &= -3 \quad \text{or} \quad x = 5 \end{aligned}$$

$$P(-3, 22) \text{ and } (5, 22)$$

Example 2:

A parabola has a vertex V(3,-2) and passes through the point P(9,16). What are the zeros of the function? The initial value?

$x \ y$

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a(x - 3)^2 - 2$$

$$16 = a(9 - 3)^2 - 2$$

$$16 = a(6)^2$$

$$\frac{16}{36} = \frac{a(36)}{36}$$

$$a = \frac{1}{2}$$

$$\therefore f(x) = \frac{1}{2}(x - 3)^2 - 2$$

Zeros:

$$-\frac{k}{a} = -\frac{-2}{\frac{1}{2}} = 4 \therefore 2 \text{ zeros}$$

$$\begin{aligned} x_1 &= 3 - \sqrt{4} & x_2 &= 3 + \sqrt{4} \\ &= 3 - 2 & &= 3 + 2 \\ &= 1 & &= 5 \end{aligned}$$

$$\text{Zeros: } x \in \{1, 5\}$$

$$\text{or } (1, 0) \text{ & } (5, 0)$$

y int: $x = 0, y = ?$

$$\begin{aligned} f(x) &= \frac{1}{2}(x - 3)^2 - 2 \\ &= \frac{1}{2}(0 - 3)^2 - 2 \\ &= 2.5 \end{aligned}$$

$$\text{y int: } (0, 2.5)$$