

Quadratic Functions - Parabolas

1) Type One - Equation in Standard Form

$$f(x) = a(x-h)^2 + k$$

a

- $a > 0$ → parabola opens up ☺
- $a < 0$ → parabola opens down ÿ
- a controls how wide (fat) or skinny a parabola is.
- If $a > 1$ → skinnier (vertically stretched)
- $0 < a < 1$ → wider (horizontally stretched)

h,k

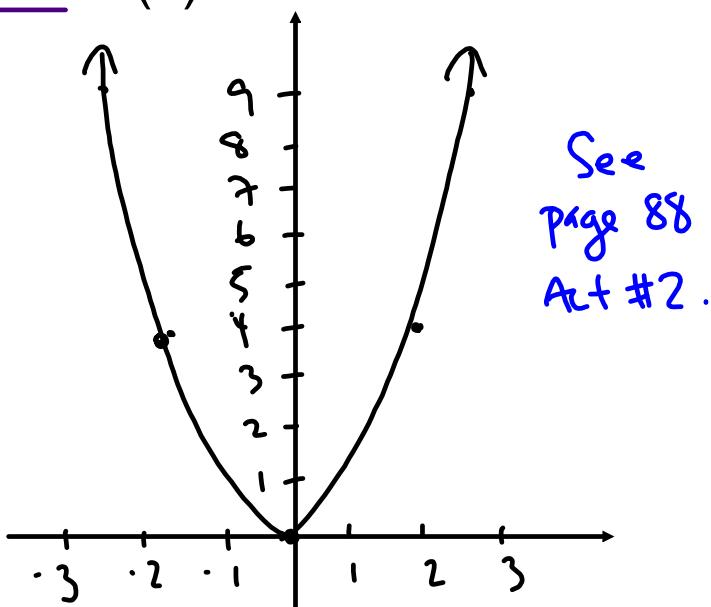
- (h,k) is the vertex of a parabola, $V(h,k)$
- h → horizontal translation of the vertex (x)
- k → vertical translation of the vertex (y)

Note: Always take the opposite sign of h

Axis of symmetry: $x = h$

Basic Parabola - $f(x) = x^2$

x	y
-3	9
-2	4
0	0
2	4
3	9



Given the following quadratic functions, state whether the parabolas open up or down, the axis of symmetry and the vertex.

a) $f(x) = (x-3)^2 + 4$

- opens \uparrow ($a > 0$)
- $x = 3$
- $V(3, 4)$

b) $f(x) = -2(x+4)^2 + 1$

- opens \downarrow ($a < 0$)
- $x = -4$
- $V(-4, 1)$

c) $f(x) = -(x-4)^2 - 1$

Finding the zeros of a function

The number of zeros can be determined by the sign of $\frac{-k}{a}$.

There are 3 cases:

- $\frac{-k}{a} > 0 \rightarrow$ there are two zeros

$$x_1 = h - \sqrt{\frac{-k}{a}} \quad x_2 = h + \sqrt{\frac{-k}{a}}$$

- $\frac{-k}{a} = 0 \rightarrow$ there is only one zero

$$x_1 = x_2 = h$$

- $\frac{-k}{a} < 0 \rightarrow$ no zeros

$$S = \emptyset \quad \emptyset \leftarrow \text{Empty set or null set}$$

Examples: Find the zeros if they exist

1) $f(x) = 2(x-1)^2 - 8$

$$a=2 \quad \text{check: } \frac{-k}{a} = \frac{-(-8)}{2} = 4 \quad \therefore 2 \text{ zeros}$$

$$h=1$$

$$k=-8$$

$$\begin{aligned} x_1 &= 1 - \sqrt{4} & x_2 &= 1 + \sqrt{4} \\ &= 1 - 2 & &= 1 + 2 \\ &= -1 & &= 3 \end{aligned}$$

$$S = \{-1, 3\}$$

2) $f(x) = 2(x-1)^2$

$$a=2 \quad \text{check: } \frac{-k}{a} = \frac{-0}{2} = 0 \quad \therefore 1 \text{ solution}$$

$$h=1$$

$$k=0$$

$$x_1 = x_2 = 1$$

$$S = \{1\}$$

3) $f(x) = 2(x-1)^2 + 8$

$$a=2 \quad \text{check: } \frac{-k}{a} = \frac{-8}{2} = -4 \quad \therefore \text{no solutions}$$

$$h=1$$

$$k=8$$

Since $\frac{-k}{a} < 0$.