

Absolute Value Function

The absolute value of a function considers only the magnitude of the number and not the sign. It is the distance away from 0.

Example: $|3| = 3$ and $|-3| = 3$

Properties of absolute value:

1. $|a| \geq 0$ ex. $|-8| \geq 0$ since $|-8| = 8$

2. $|m| = |-m|$ ex. $|10| = |-10| = 10$

3. $|a \times b| = |a| \times |b|$ ex. $|4(x-2)| = |-8| = 8$
 $|4(x-2)| = 4|x-2| = 4 \cdot 2 = 8$

4. $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$ ex. $\left| \frac{9}{-3} \right| = \frac{|-3|}{|3|} = 3$
 $\frac{|9|}{|-3|} = \frac{9}{3} = 3$

5. $|a + b| \leq |a| + |b|$ ex. $|10 + (-5)|$
 $= |5| = 5$
 $|10| + |-5|$
 $= 10 + 5 = 15$

6. $|a - b| \geq |a| - |b|$ ex.
 $|10 - (-5)| = |15| = 15$
 $|10| - |-5| = 10 - 5 = 5$
 $15 \geq 5$

Absolute Value Equations

The number of solutions to the equation $|x| = k$ depends on the sign of k .

If $k > 0$, the equation has 2 solutions.

$$x = -k \text{ or } x = k$$

Ex. $|x| = 4$ $S = \{-4, 4\}$

If $k = 0$, the equation has 1 solution.

$$x = 0$$

Ex. $|x| = 0$ $S = \{0\}$

If $k < 0$, the equation has no solutions

Ex. $|x| = -5$ $S = \emptyset$

Solve, if possible:

a) $|3x - 5| = 12$

$$\begin{array}{l} 3x - 5 = 12 \\ 3x = 17 \\ x = \frac{17}{3} \\ x \in \left\{-\frac{7}{3}, \frac{17}{3}\right\} \end{array}$$

b) $\frac{3|x - 5|}{3} = 12$

$$\begin{array}{l} |x - 5| = 4 \\ x - 5 = -4 \\ x = 1 \\ x \in \{-9, 1\} \end{array}$$

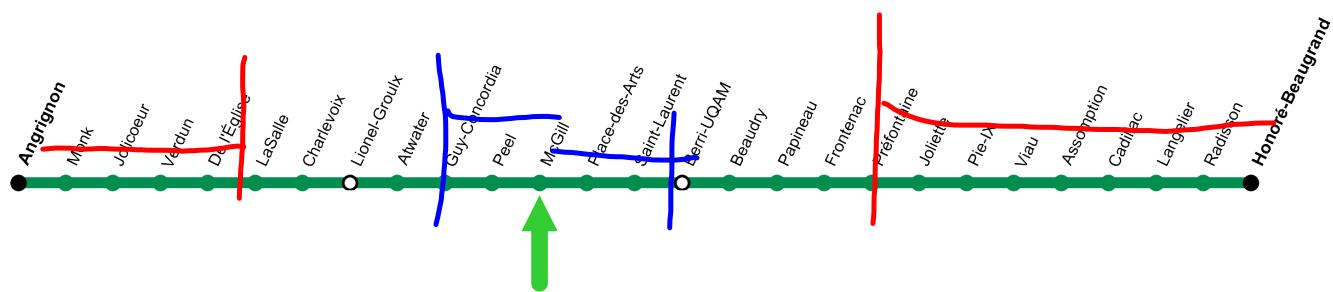
c) $-2|x + 7| = 8$

$$\begin{array}{l} |x + 7| = -4 \\ x \notin \emptyset \end{array}$$

d) $|3x + 8| - 2 = 11$

$$\begin{array}{l} |3x + 8| = 13 \\ 3x + 8 = -13 \quad 3x + 8 = 13 \\ \frac{3x}{3} = -\frac{21}{3} \quad \frac{3x}{3} = \frac{5}{3} \\ x = -7 \quad x = \frac{5}{3} \\ x \in \left\{-7, \frac{5}{3}\right\} \end{array}$$

- "I lied, I was actually **more** than 6 stops away"



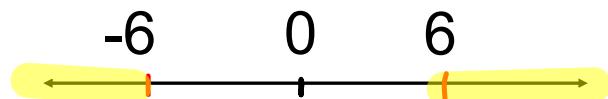
- "But no worries, now I'm **less than three stops** from McGill!"

Absolute Value Inequalities

Solve:

$$\begin{aligned} |x| &\geq 6 \\ |x| &= 6 \\ x &\in \{-6, 6\} \end{aligned}$$

$|x| \geq k$
 $x \leq -k \text{ or } x \geq k$
 $S =]-\infty, -k] \cup [k, +\infty[$

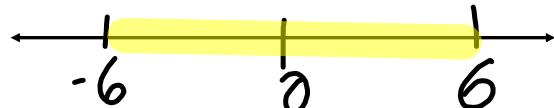


$$x \in]-\infty, -6] \cup [6, +\infty[$$

$$\begin{aligned} |x| &\leq 6 \\ |x| &= 6 \end{aligned}$$

$$x \in \{-6, 6\}$$

$|x| \leq k$
 $x \geq -k \text{ and } x \leq k$



$$x \in [-6, 6]$$

Solve, if possible:

e) $-4|x + 3| + 1 \leq 0$ f) $|2x - 3| \leq -7$

$$\begin{aligned} -4|x + 3| + 1 &= 0 \\ -4|x + 3| &= -1 \end{aligned}$$

$$\begin{aligned} |x + 3| &= \frac{1}{4} \\ x + 3 &= \frac{1}{4} \quad \mid x + 3 = -\frac{1}{4} \\ x &= -\frac{11}{4} \quad \mid x = -\frac{13}{4} \end{aligned}$$

$x \in]-\infty, -\frac{13}{4}] \cup [-\frac{11}{4}, +\infty[$

g) $|2x - 3| \geq -7$

$$|2x - 3| \geq 0$$

$$x \in \mathbb{R}$$