Properties of Radicals

- If $a \in \mathbb{R}_+$ and $b \in \mathbb{R}_+$,

  $\sqrt{a} = b \Leftrightarrow b^2 = a$ ex. $\sqrt{25} = 5$ since $5^2 = 25$

- If $a \in \mathbb{R}_+$ and $b \in \mathbb{R}_+$, we have the following:

① $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ ex. $\sqrt{16 \times 9} = \sqrt{16} \times \sqrt{9}$

② $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$ ex. $\sqrt{\frac{16}{100}} = \frac{\sqrt{16}}{\sqrt{100}}$

③ $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ ex. $\sqrt{64+36} \neq \sqrt{64} + \sqrt{36}$

④ $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$ ex. $\sqrt{100-64} \neq \sqrt{100} - \sqrt{64}$

⑤ $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$ ex. $2\sqrt{3} \times 5\sqrt{2} = 10\sqrt{6}$

⑥ $a\sqrt{b} \div c\sqrt{d} = \frac{a}{c}\left(\sqrt{\frac{b}{d}}\right)$ ex. $12\sqrt{10} \div 4\sqrt{2}$
$= 3\sqrt{5}$

Examples:

① $\sqrt{16} \times \sqrt{36} = 4 \times 6$
$= 24$

② $\sqrt{8} \times \sqrt{2} = \sqrt{16}$
$= 4$

③ $4\sqrt{2} \times \sqrt{5} = 4\sqrt{10}$