Proving a Trigonometric Identity

Method:

- Simplify one side, usually the more complicated side, to make it identical to the other side

It's helpful to know (and use!):

- Basic Identities
- Trig Ratios (tan, csc, cot, sec)
- Factoring, adding and subtracting rational expressions, multiplication and division
- And a little bit of imagination

Prove:

1) sinx + cosx cotx = cscx

$\sin x + \cos x\left(\frac{\cos x}{\sin x}\right)$

$\left(\frac{\sin x}{\sin x}\right)\sin x + \frac{\cos^2 x}{\sin x}$

$\frac{\sin^2 x + \cos^2 x}{\sin x}$

$\frac{1}{\sin x}$

$\csc x = \csc x$ Q.E.D

2) $\sin\theta \tan\theta + \sec\theta = \frac{\sin^2\theta + 1}{\cos\theta}$

$\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) + \frac{1}{\cos\theta}$

$\frac{\sin^2\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$\frac{\sin^2\theta + 1}{\cos\theta} = \frac{\sin^2\theta + 1}{\cos\theta}$

3) $\frac{1 - \tan x}{1 - \cot x} = -\tan x$

$\frac{1 - \frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}}$

$\frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}}$

$\frac{\cos x - \sin x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x}$

$\frac{-(\cancel{\sin x - \cos x})}{\cos x} \cdot \frac{\sin x}{\cancel{\sin x - \cos x}}$

$-\frac{\sin x}{\cos x}$

$-\tan x = -\tan x$

4) $\sin^2 x \cot^2 x = 1 - \sin^2 x$

$\cancel{\sin^2 x}\left(\frac{\cos^2 x}{\cancel{\sin^2 x}}\right)$

$\cos^2 x$

$1 - \sin^2 x = 1 - \sin^2 x$

5) $\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x}$

$\frac{\frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$

$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$

$\frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{1}$

$\sin^2 x = \sin^2 x$