

Trigonometric Identities

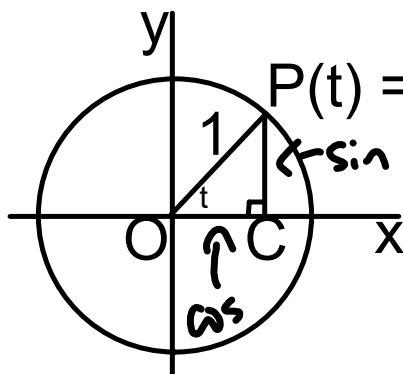
Basic Identities

$$1) \sin^2 t + \cos^2 t = 1$$

$$2) 1 + \tan^2 t = \sec^2 t$$

$$3) 1 + \cot^2 t = \csc^2 t$$

Prove: $\sin^2 t + \cos^2 t = 1$



$$P(t) = (\cos t, \sin t)$$

$$\sin t = \overline{CP}$$

$$\cos t = \overline{OC}$$

$$\overline{CP}^2 + \overline{OC}^2 = 1 \quad (\text{by pythagoras})$$

$$\sin^2 t + \cos^2 t = 1 \quad \checkmark$$

Reduce to a single term:

$$1) 1 - \cos^2 t = \sin^2 t \quad (\text{since } \sin^2 t + \cos^2 t = 1)$$

$$2) \sec^2 t - \tan^2 t = 1 \quad (\text{since } 1 + \tan^2 t = \sec^2 t)$$

$$3) \cot x \cdot \sec x = \frac{\cancel{\cos x}}{\sin x} \cdot \frac{1}{\cancel{\cos x}} = \frac{1}{\sin x} = \csc x$$

$$4) \left[\csc^2 x - \cot^2 x \right] - \sin^2 x \quad (\text{since } 1 + \cot^2 x = \csc^2 x)$$

$$1 - \sin^2 x$$

$$\cos^2 x$$