#1

$f(x)=x-2$

$g(x)=\frac{5x}{x-3}$

Find $g \circ f$ in standard form

$(g \circ f)(x) = g(f(x))$

$= \frac{5(x-2)}{(x-2)-3}$

$= \frac{5x-10}{x-5}$

$= \frac{15}{x-5} + 5$

Find $f \circ g$ in general form

$f(g(x)) = \left(\frac{5x}{x-3}\right) - 2$

$= \frac{5x}{x-3} - \frac{2x-6}{x-3}$

$= \frac{3x+6}{x-3}$

#2

$f(x)= -\sqrt{8-4x} + 2.$

When is $f(x)>-2$?

$-\sqrt{8-4x}+2 > -2$

$-\sqrt{8-4x} > -4$

$\sqrt{8-4x} < 4$

$8-4x < 16$

$-4x < 8$

$x > -2$

$8-4x \geq 0$

$-4x \geq -8$

$x \leq 2$

$\therefore x \in]-2, 2]$

#3

$f(x)=x^2-2x+1$

$g(x)=x+1$

$h(x)=\sqrt{x}-1$

Find $(h \circ f \circ g)(2)$

$(f \circ g)(x) = (x+1)^2 - 2(x+1) + 1$

$= x^2 + 2x + 1 - 2x - 2 + 1$

$= x^2$

$(h \circ f \circ g)(x) = h(f(g(x)))$

$= \sqrt{x^2} - 1$

$= x - 1$

$(h \circ f \circ g)(2) = 2 - 1 = 1$

Exponential Functions

Keywords: doubles, triples, halves, % increase/decrease.

$$f(x)=ac^{bx}$$

a = initial amount

c = rate of increase/decrease*

b = number of times *c* is applied in a given time period

*Note: c can be given as a number, or as a decimal/ percentage:

Number: doubles (c=2), triples (c=3), halves (c=1/2)

Percentage: 1±% increase/decrease:

- A debt increases by 10% each month:
  c=1+10%=1+0.1=1.1
- A car loses 2.5% of its value each month:
  c=1-2.5%=1-0.025=0.975

Examples: identify x, y, a, b, c, and write the rule

1. A petri dish has 50 bacteria initially. The number of bacteria triples every hour.

x: time (hours)
y: number of bacteria
⇒f(x)= $50(3)^x$

a= 50
c= 3
b= 1

2. A frog pond has an initial population of 25 frogs. The population quadruples twice a year.

x: time (years)
y: number of frogs
⇒f(x)= $25(4)^{2x}$

a= 25
c= 4
b= 2

3. There are initially 100 bacteria in a sample. The bacteria double every 20 minutes.

x: time (hours)
y: number of bacteria
⇒f(x)= $100(2)^{3x}$

a= 100
c= 2
b= $60 \div 20 = 3$

4. An initial population of 2000 penguins increases by 18% every year.

x: time (years)
y: number of penguins
⇒f(x)= $2000(1.18)^x$

a= 2000
c= $1 + 0.18 = 1.18$
b= 1

5. A car is purchased for $40,000 and loses 17% of its value every year.

x: time (years)
y: value of the car ($)
⇒f(x)= $40000(0.83)^x$

a= 40 000
c= 0.83
b= 1

6. An initial population of 150 hippos increases by 6% every 2 years.

x: time (years)
y: number of hippos.
⇒f(x)= $150(1.06)^{\frac{x}{2}}$

a= 150
c= 1.06
b= $\frac{1}{2}$

Finding the Rule from a Table of Values

$y=ac^x$ (only)

Example:

x	0	2
y	2	8

$$\begin{cases} 2 = ac^0 \\ 8 = ac^2 \end{cases}$$

$$\begin{cases} 2 = a(1) \\ 8 = ac^2 \end{cases}$$

$a = 2$

$\therefore 8 = 2c^2$

$4 = c^2$

$c = \pm 2$

$c > 0$

$\therefore c = 2$

$y = 2(2)^x$